Teacher Note

Place Value

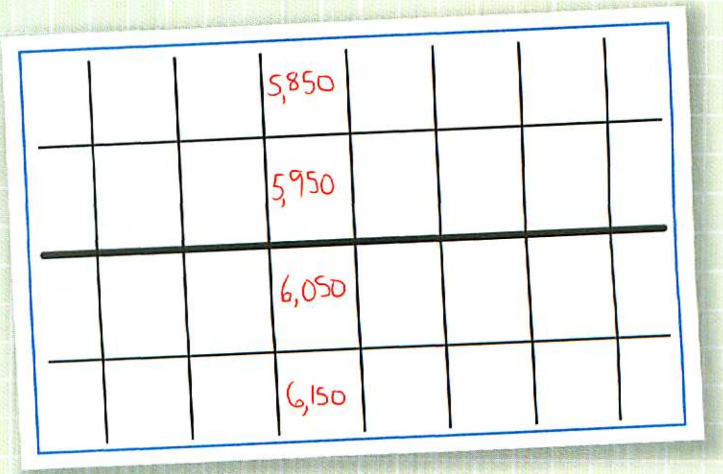
The base-ten number system is a place-value system; that is, any numeral, such as 2, can represent different values, depending on where it appears in a written number: it can represent 2 ones, 2 tens, 2 hundreds, 2 thousands, and so on. Understanding our place-value system requires coordinating the way we write the numerals that represent a particular number (e.g., 5,217) and the way we name numbers in words (e.g., five thousand, two hundred seventeen) with how those numerals represent quantities. (See **Place Value** in *Implementing Investigations in Grade 5.*) In Grade 4, students learned to use and understand numbers in the thousands. In this unit, students revisit their work on numbers up to 10,000 and expand their work to even larger numbers.

The Base-Ten Number System

The heart of the work on place value in Grade 5 is relating the written numerals to the quantity and to how the numerals are composed. Being able to do this is not simply a matter of saying that 5,217 "has 5 thousands, 2 hundreds, 1 ten, and 7 ones," which we know students can easily learn to do without attaching meaning to the quantity these numerals represent. Students must learn to visualize how 5,217 is built up from thousands, hundreds, tens, and ones in a way that helps them relate its value to other quantities. Understanding the place value of a number such as 5,217 entails knowing that 5,217 is closer to 5,000 than to 6,000; that it is 1,000 more than 4,217, 100 more than 5,117, 17 more than 5,200, and 3 less than 5,220; and that it can be decomposed in a number of ways, such as 52 hundreds, 1 ten, and 7 ones.

In this unit, students use 10,000 charts to visualize numbers in the thousands and their relationships. From their work in Grades 3 and 4, students should have a solid understanding of how 1,000 is composed of ten 100s and how each 100 is composed of ten 10s. In Grade 4, students worked with a class 10,000 chart composed of one hundred 100 charts. By building up to 1,000 and then 10,000 from the 100 charts, with which they were very familiar, they learned how these larger numbers are composed. However, because the chart was built from individual 100 charts, the numbering system of each 100 was contained within an individual chart.

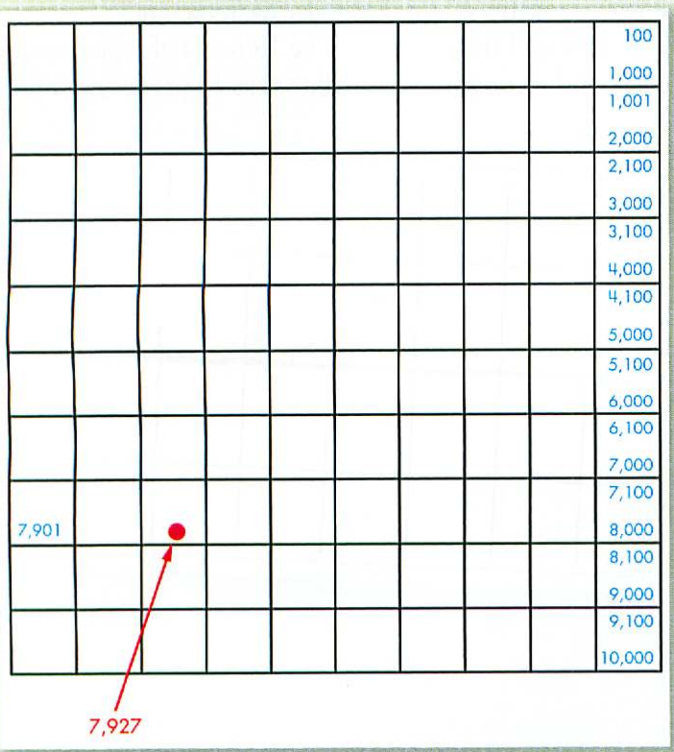
In Grade 5, we introduce a different kind of 10,000 chart, in which the dimensions and the numbering system provide a visual image of 10,000 in various ways. The 100 rows with 100 squares in each row provide an image of how 10,000 is composed of one hundred 100s. Each row of the chart contains 100 squares and the rows are numbered 1–100, 101–200, 201–300, and so on. Each group of 10 rows is outlined as a rectangle. Each rectangle contains 10 × 100, or 1,000, squares. The 10 rectangles provide an image of the composition of 10,000 as 10 groups of 1,000.



Sample Student Work

Some place-value models, such as base-ten blocks, use three dimensions to help students visualize thousands. We have chosen to continue building on the flat model that students have been using throughout the grades, starting with the 100 chart, which they can now visualize well. In Grade 5, they use new units—rows of 100 and rectangles of 10 hundreds—as the building blocks for composing 10,000.

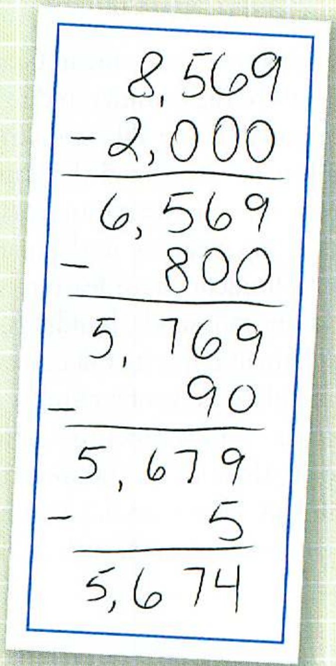
By using this flat model composed of 100 hundreds, students see all 10,000 squares arranged in a way that helps them visualize the structure of the base-ten system. Through placing numbers on the chart, they consider relationships between numbers. For example, about where would the number 7,927 be? To place this number, students bring into play their knowledge of the relationship of this number to 10,000, to 8,000, to 7,900, and to 7,930. In this process, they are associating the written number with its meaning: the number is in the eighth rectangle, the one that contains the squares numbered 7,001 to 8,000. Because the number is 900 more than 7,000, it is found in the last row of that rectangle, the row that starts with 7,901 and ends with 8,000. It is about a quarter of the way along that row—the 27th square.



Place Value and Computational Fluency

Students' work on addition and subtraction relates directly to their work on the place-value system. Understanding the place value of each digit in a number leads to an understanding of the magnitude of the number and allows students to estimate what a reasonable answer should be before they carry out any computation. Developing the habits of estimating before computing and of comparing the solution with the estimate is important for all students.

Efficient strategies for solving addition and subtraction problems depend on knowing the addition combinations and their subtraction counterparts and understanding how to add or subtract multiples of 10, 100, 1,000, and so on. Developing fluency with addition and subtraction requires understanding the quantity that digits in any place represent. For example, many students subtract in parts for a problem such as 8,569 − 2,895.



Sample Student Work

In order to carry out this computation, students draw on an understanding of the meaning of each digit. For example, the 2 represents 2,000. Further, they use knowledge about the equivalencies of 1,000 and ten 100s, one 100 and ten 10s, and so on. For example, in the second step of the problem, this student thinks of the first two digits of 6,569 as representing 65 hundreds rather than as 6 thousands and 5 hundreds, allowing the student to subtract 8 hundreds.

Students are also using their knowledge of place value and the basic subtraction "facts" to easily subtract large numbers: $13 - 7 = 6$ (6 ones), $130 - 70 = 60$ (6 tens), $1{,}300 - 700 = 600$ (6 hundreds). Subtracting 7 from 13 in any place is the same, except that the units that are subtracted are ten times larger in each successive place to the left. In Grade 5, students should be adding or subtracting the largest possible chunks of numbers, rather than counting by 100 or 1,000. Similarly, students are applying other computations they can carry out mentally with numbers below 100 to problems with larger numbers. In the problem above, being able to mentally compute $65 - 8 = 57$ (by subtracting 5 and then 3, for example, or just knowing that $15 - 8 = 7$, so $65 - 8 = 57$) is applied to subtracting 8 hundreds from 65 hundreds.

The strategies for addition used by many students—adding by place or adding on one number in parts—depend on an understanding of how to decompose numbers. (See **Teacher Note:** Addition Strategies on page 116.) The common subtraction strategies of subtracting in parts, adding up, or subtracting back also depend on an understanding of how to decompose numbers. (See **Teacher Note:** Subtraction Strategies on page 119.) The U.S. algorithm for subtraction, which students study in Investigation 2, also depends on a firm grasp of place value.